9. G. A. COOPER. Ph.D. Thesis Cambridge University (1966).
10. J. MORTON and G. W. GROVES, *J. Mater. Sci.* 9 (1974) 1436.

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Spherical and cylindrical models for craze growth

In a previous paper [1] we applied the technique of finite element analysis to the problem of failure by yielding of a uniform cylindrical void system, taken as a model for the process of craze growth in plastics. However, the smallest voids formed during the initiation of crazing often seem to have a spherical rather than a cylindrical form [2, 3]. We have, therefore, repeated some of our previous calculations using a spherical hole model to establish whether or not there is a significant difference in the conclusions reached. The new model simply substitutes a three-dimensional system of spherical holes for the two-dimensional cylindrical voids used in the previous paper [1].

A single module of the spherical void array is illustrated in Fig. 1 where the voids are of radius a and their centres are spaced at an equal distance of $2(a + d)$ in each of the x, y and z directions. The loading of prime interest is a hydrostatic tensile loading which can be accomplished by prescribing displacements Δ on the three faces $x = a + d, y = a + d$ and $z = a + d$ in the x, y and z directions respectively. The three faces defined by $x = 0$, $y = 0$ and $z = 0$ are restrained from moving in the x, y and z directions respectively.

Initially, two void volume fractions were considered corresponding to $d/a = 0.5$ and $d/a = 0.1$. In each case the solution was performed using 1, 8 and 27 three-dimensional quadratic elements in turn and the 8 element mesh employed is illustrated in Fig. 1b. A Von Mises yield criterion was assumed and the material properties, listed in Fig. 1a, are the same as those employed in [1]. The variation of the total reactive force on any face with increasing prescribed displacement is shown in Fig. 2a

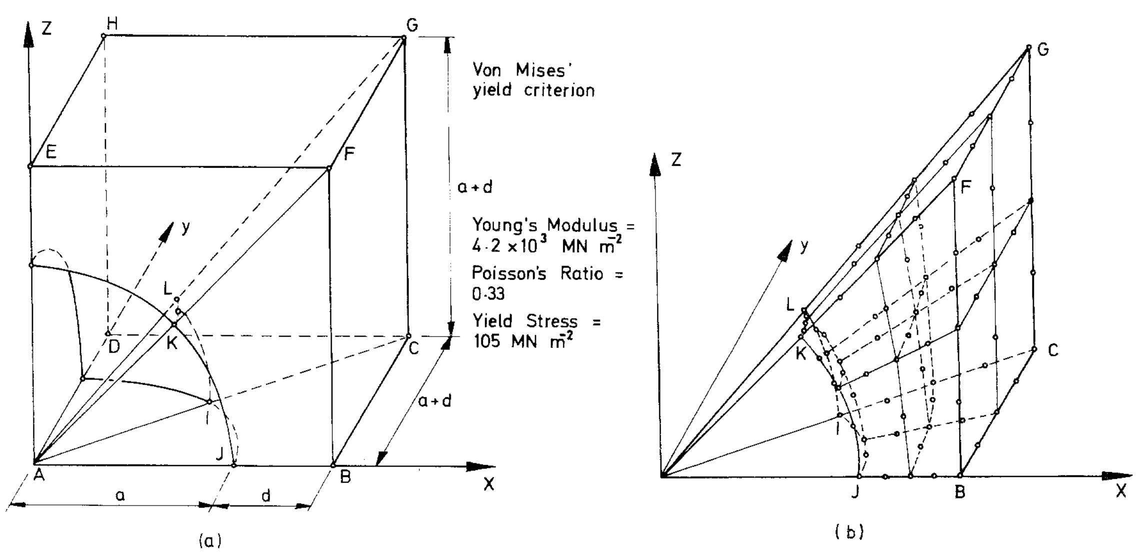
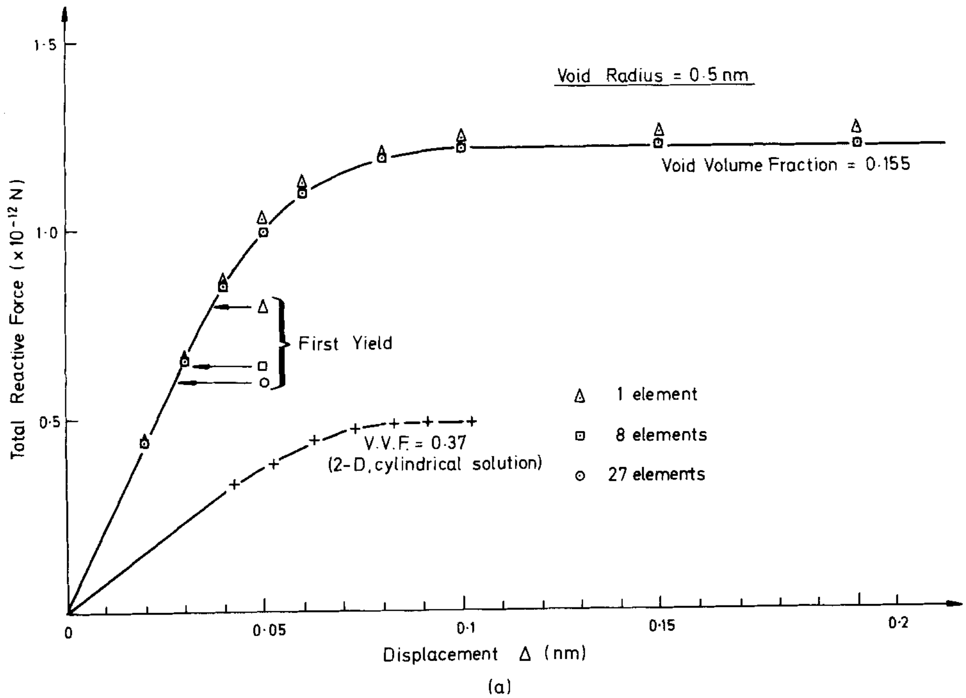


Figure 1 Illustration of spherical void model. (a) Module analysed; (b) quadratic isoparametric element mesh employed.



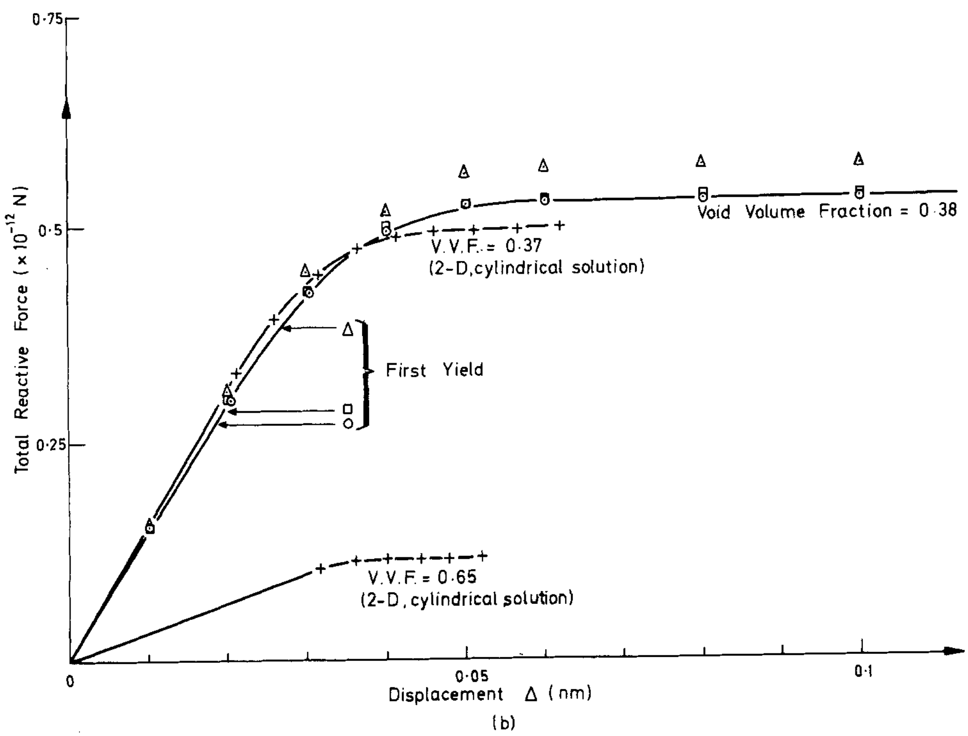


Figure 2 Calculated stress/displacement curves for cylindrical and spherical models (for two-dimensional solution with cylindrical model).

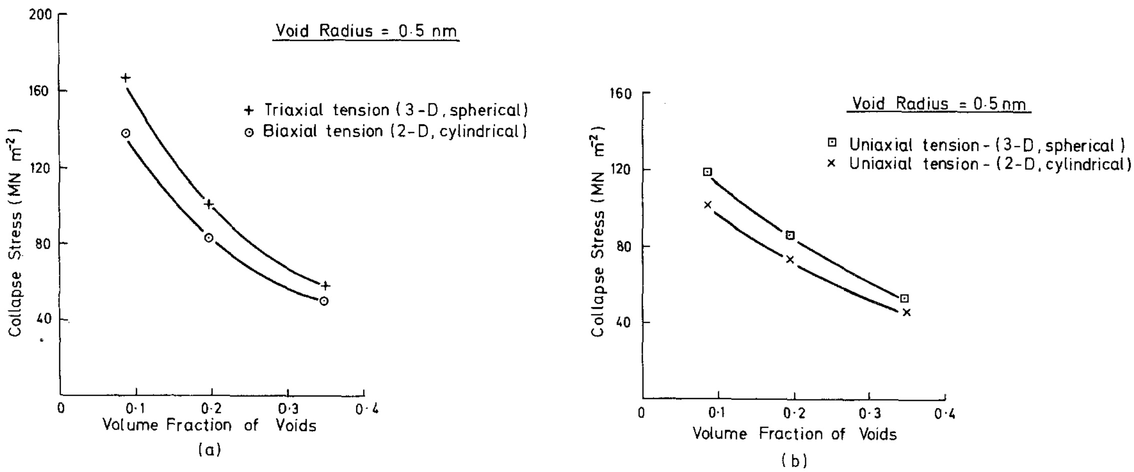


Figure 3 Collapse stress plotted against volume fraction of voids for cylindrical and spherical models. (a) Hydrostatic tensile loading; (b) uniaxial tension–fixed edge. Points for spheres to be placed on the relevant curve for Fig. 5 of previous paper [1].

and b for $d/a = 0.5$ and $d/a = 0.1$ respectively. For both situations the results for the 8 and 27 element meshes are in good agreement, with the 1 element idealization over-estimating the stiffness of the model. The results of the previous two-dimensional investigation [1] with the same d/a values are also included in Fig. 2a and b. The two-dimensional void volume fraction of 0.37 in Fig. 2a and 0.65 in Fig. 2b correspond to the same void size and spacing used in the respective three-dimensional analyses and it is evident that on this basis a large discrepancy exists between the two-dimensional and three-dimensional results. However, from Fig 2b it is seen that the two-dimensional and three-dimensional solutions compare well when the void volume fraction is chosen to be of comparable value, the collapse loads by both analyses then only differing by some 5%.

Also of importance is the loading of such models in uniaxial tension with a restrained lateral edge condition as considered for the cylindrical model in [1]. A plot of collapse stress against void volume fraction is shown in Fig. 3. In Fig. 3a two- and three-dimensional results for hydrostatic loading are compared for three of the void volume fractions employed in the previous paper [1] and Fig. 3b shows a similar comparison for the uniaxial loading case.

From these results we can conclude that on a volume basis the differences in yield behaviour in hydrostatic tension between the spherical and cylindrical models are not large, so that further calculations may safely be made with the cylindrical model.

References

1. R. N. HAWARD and D. R. J. OWEN, *J. Mater. Sci.* 8 (1973) 1136.
2. R. P. KAMBOUR, *Polymer* 5 (1964) 143.
3. P. BEAHAN, M. BEAVIS and D. HULL, *Phil. Mag.* 24 (1971) 1267.

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